THURSDAY, MARCH 20, 1902.

REFORM OF THE TEACHING OF MATHEMATICS.

British Association Meeting at Glasgow, 1901. *Discussion on the Teaching of Mathematics, which took place on September* 14 *at a Joint Meeting of two Sections, Section A, Mathematics and Physics, Section L, Education.* Edited by John Perry. Pp. vi + 101. (London: Macmillan and Co., Ltd.) Price 2*s.* net.

PROF. PERRY'S views concerning the teaching of mathematics have been expounded often in the columns of NATURE and elsewhere, and have aroused a great deal of interest. It was a happy idea to bring about a discussion of these views at a joint meeting of the mathematical and educational Sections of the British Association. The report of that discussion has been issued as a separate small volume. It contains the introductory address delivered by Prof. Perry, a specimen syllabus prepared by him for use in training colleges, the discussion that followed the address, written remarks on the subject communicated by several teachers who did not take part in the discussion, and Prof. Perry's reply. The discussion was restricted to what may be called a possible school range of mathematics; the teaching of advanced mathematics at the Universities was scarcely touched upon. As an outcome of the discussion, a committee of the British Association was appointed

> "to report upon improvements that might be effected in the teaching of mathematics, in the first instance of elementary mathematics, and upon such means as they think likely to effect such improvements."

The movement thus initiated is important. Many of the more enlightened teachers of mathematics in Britain have long been dissatisfied with the conditions under which they have been compelled to work; and efforts in the direction of reform have been made, in particular by the Association for the Improvement of Geometrical Teaching. The smallness of the results achieved by those efforts is to be traced mainly to the action of the chief examining bodies; but there are not wanting signs that these bodies are now less complacently conservative than they have been in the past. It is earnestly to be hoped that the indicated change of attitude may turn out to be real. The handy publication under notice should be of great assistance to those who are agitating for reform.

The movement is in great part a reaction against the pedantic and unpractical character of the habitual teaching of mathematics in schools. This character belongs both to the methods of presenting particular subjects and to the order in which the subjects, and the parts of the subjects, are studied. The methods and the order now in vogue are not, of course, a system devised purposely; they have been arrived at gradually, and are sanctioned by tradition. The principles of the proposed reform may be stated broadly as follows:—(1) The way for every abstract generalization should be prepared by intelligent practice of comparatively concrete processes, chosen so as to stimulate inquiry; (2) whenever a choice of alternative orders is offered, that order should be preferred which conduces best to the application of mathematics to practical needs. A subsidiary principle of reform would be that all purely artificial and unimportant developments should be omitted. The principles thus stated will not perhaps excite much opposition; yet far-reaching and radical changes would be required to give effect to them. One result that may be expected to follow from such changes would be that the study of the elements of mathematics would become a better preparation than it is at present for the study of the more advanced theories. The mathematics of most of our elementary text-books is felt to be almost as much out of touch with modern mathematics as with everyday life. This conviction has had some share in promoting the movement in favour of change.

As regards geometry, the thoroughgoing adoption of the above-stated principles of reform would lead in the first place to the introduction of courses of practical work in drawing and measuring, which would precede and accompany the study of demonstrative geometry; and it would lead in the second place to a great simplification of that study. That simplification would consist partly in the abandonment of Euclid's "Elements" as a text-book. The difficulty of finding a generally acceptable substitute for Euclid was fatal to the proposals of the Association for the Improvement of Geometrical Teaching, and the controversy between the supporters and the opponents of Euclid has been a main feature of all discussions concerning the improvement of the teaching of mathematics in this country. Now it may be contended that we ought not to attempt to present geometry to children as a formal system deduced from a minimum number of assumptions; but that our objects should be to impart knowledge of the properties of geometrical figures and to cultivate power of geometrical reasoning. Upon the general adoption of this view of the true objects of geometrical teaching, the above-mentioned difficulty would disappear; for it is safe to assert that these objects can be secured better without Euclid's book than with it. In this matter the French are far ahead of us. Their text-books of geometry have been evolved by a natural process of development from the "Géométrie" of Legendre; and text-books similar to those of France are in use in Germany, Italy and the United States. It is far from my purpose to suggest that British teachers should now adopt as a substitute for Euclid a translation of a French book or any other foreign book. In all these books too much attention, as I think, is paid to the development of a formal logical system, with the result that much space is occupied in proving propositions that are sufficiently obvious without proof. We have yet to work out for ourselves the method that is best suited to our educational needs and national temperament. This cannot be done without freedom from the tyranny of an orthodox standard; nor can it be done in a hurry. Changes will have to be made gradually. In this connexion we may note that the memorial sent by some schoolmasters to the committee of the British Association (NATURE, January 16, 1902, p. 233) points out a promising method of beginning to make changes. If the chief examining bodies would now sanction such a departure from existing practice as is there proposed, progressive reform would become possible.

© 1902 Nature Publishing Group

The contention that Euclid's "Elements" is an unsuitable text-book is borne out by a critical scrutiny. The book was an attempt to deduce a complete system of geometry from a minimum number of assumptions, contained in definitions, axioms and postulates. No such attempt, even if completely successful, can possibly appeal to immature minds; and it is well known that Euclid's attempt, in spite of its many conspicuous merits, is logically defective. This would not matter very much if it were otherwise well adapted to effect the two objects which we have noted above as the true objects to be aimed at in teaching geometry. Notoriously it is not so adapted; striking instances of its deadening effect upon the minds of average boys have been recorded by Prof. Minchin and others. Such instances may fairly be cited in opposition to the claim, urged by the supporters of Euclid, that his system affords valuable mental training; and this claim does not gain in force when it is observed that most of the men who have undergone the training appear to be unable to appreciate the logical defects of the system, and that they accept without question the absurdities that are often made to do duty, in our text-books of analysis and mechanics, as definitions or as proofs. Further, the tacit agreement to drop Euclid's fifth book has robbed his sequence of its significance. This book is really the great contribution of antiquity to the problem of irrational numbers—perhaps the central problem of mathematics. It is certainly the keystone of Euclid's system. Without it, there is no reason why proportion and measurement, treated arithmetically, should not take a much earlier place in a system of geometry. In elementary stages, it will probably be best to postpone all questions of incommensurable magnitudes and irrational numbers; but sooner or later, in the training of a mathematician, such questions must be faced. The Greeks approached arithmetic through geometry, and for two thousand years Euclid's fifth book was the only theory of irrational nnmbers. Now, however, the Greek way is not the only way, nor, as I believe, the most excellent. There exists now a complete arithmetical theory of irrational numbers; and the theory of exact measurement of incommensurable, as well as of commensurable, magnitudes can be founded on a secure arithmetical basis. This revolution of mathematical thought was accomplished in Germany in the second half of the nineteenth century; its full effect has yet to be felt.

But geometry is not the only branch, even of elementary mathematics, of which the aspect would be changed entirely by giving effect to the above-stated principles of reform; nor is the need for improvement confined to the teaching of elementary mathematics. In algebra, for example, the changes advocated in the book under notice, or those proposed in the memorial of some schoolmasters already referred to, would constitute great improvements in the method of presenting the subject. Here again the French text-books are much better than most of ours. In analysis generally, the traditional order of the topics stands in need of drastic alteration—boys and girls ought not to be taught to expand the circular functions in infinite products before they have ever plotted a graph or differentiated the simplest expression. Prof. Perry has been a constant advocate of graphic methods and of the early study of the differential and integral calculus. He holds that this study should precede that of many things which now come before it, *e.g.* advanced analytical geometry of conics; this view is put forward on the ground that such an order would be more helpful to those students who will afterwards require to use mathematics in the practice of their professions. They are not the only students who would gain by the change. The educational value of mathematics would be increased enormously. Mathematics becomes an instrument of liberal education, not merely by the practice of processes—they are means to an end—nor yet by the storing of information, though knowledge of facts is an element of culture—but by the formation of exact ideas as regards both the definiteness of its fundamental notions and the inevitableness of its results. The impression that is made upon the mind, when one realises the inexorable necessity of the conclusion of a chain of reasoning, is the element in mathematical training that has been emphasized the most by those who support the claim of mathematics to be considered an integral part of a liberal education; but it may be held that the illumination to be derived from any of the fundamental notions of mathematics—such notions, for example, as proportion, continuity, vector, group—when they are thoroughly grasped, is a not less important element from the same point of view. The traditional order of study has tended to obscure the fundamental notions and the general drift of the, arguments under a cloud of secondary developments. That the differential and integral calculus can be presented at such a stage as that indicated by Prof Perry, in a manner so practical as to suit the student of engineering, and at the same time so rigorous and so luminous as to be a worthy means of liberal education, has been shown by more than one recent treatise. The examinations that have most influence upon the order of study near this stage are probably those for entrance scholarships at the colleges of the older Universities. A few reforming tutors might now initiate a change that would produce a very great effect.

It is unnecessary here to follow out the application of the principles of reform to mechanics, or to the remaining subjects of a school course of mathematics. The book under notice contains numerous suggestions on these heads. Some readers may be inclined to think that in the introductory address, and in some of the subsequent speeches, undue prominence was given to the needs of students who are destined to become engineers, or teachers in primary schools. A view that is held widely, but has not perhaps been emphasized sufficiently, is that, in the stage of ordinary school work, the course that is most suitable for such classes of students is also precisely the best for those for whom mathematics is meant to be a means of culture and for those who have the ability, and will afterwards have opportunities, to assist in the development of mathematical theories. To bring such a course into general use will require much persistent effort, directed continually to one end; and the first step will necessarily be the conversion of examiners and of the bodies that make regulations for the conduct of examinations. The future of mathematical teaching in this country is in their hands. A. E. H. L.

© 1902 Nature Publishing Group